Algebra 2

Lesson 12.3 The Fundamental Counting Principle; Permutations and combinations

Obj: Use permutations and combinations to find the number of outcomes

Fundamental Counting Principle- If event M can occur in m ways and event N can occur in n ways, then total amount is $m \times n$

Independent events.

Ex1. A sandwich menu offers customers a choice of white, wheat, or rye bread with one spread chosen from butter, mustard, or mayonnaise, and one meat chosen from turkey or ham. How many different combinations of bread and spread and meat are possible? Use tree diagram.

$\frac{3}{\text{bread}} \times \frac{3}{\text{spread}} \times \frac{2}{\text{meat}} = 18$ combinations

You try. For their vacation, the Murray family is choosing a trip to the beach or to the mountains. They can select their transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation?

$\frac{2}{\text{where}} \times \frac{3}{\text{how}} = 6$

BC mc
BP mP
BT mT

You try. Many cell phones allow owners to call and get their messages by entering a 4 digit code. How many codes are possible?

4 digits

$\frac{10}{0\text{-}9 \text{ choose}} \times \frac{10}{} \times \frac{10}{} \times \frac{10}{} = 10^4 = 10{,}000$

Dependent Events (permutations)

Ex2: Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have? each time choices reduce

Period	1st	2nd	3rd	4th	5th	6th
Number of Choices	6	5	4	3	2	1

↑ choices ↑ choices ↑

→ $6!$

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

720

You try: each time here choice reduces

Gabriela is making a playlist with her 3 favorite songs. How many possible orders are there for the songs?

3 × 2 × 1 = 6

This is called a permutation. A permutation is: an arrangement in which order matters.

The number of permutations is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

think pick pres, VP, secretary

n = total items r how many you choose/use

You do not have to choose all of your items.

What is a factorial? $n!$ means multiply all numbers upton

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

Ex3. Gabriela wants to make another playlist with five her favorite artist's eight most popular songs. How many different playlists are possible?

8 total choose 5 $\quad _8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$

6720

You may use the fun counting principle or a permutation formula.

or 8 × 7 × 6 × 5 × 4 choices

Combinations: an arrangement in which order does not matter.

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$ ← takes out repeats

think picking a team of 5 or committee

Ex 4. Marisol is planning to be a counselor at summer camp. She can choose 3 activities for her session. There are 10 activities available. How many different combinations of 3 activities are possible?

$${}_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = \frac{720}{6} = \boxed{120}$$

You try. Decide if each is a permutation, combination, fun counting principle and solve.

1. Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes one student. In how many ways can ten time slots be assigned? order matters perm

$${}_{15}P_{10} = \frac{15!}{5!} \approx 1.089 \times 10^{10}$$

$$10,897,286,400$$

2. You have five shirts and four pairs of pants. How many different ways can you arrange your shirts and pants into outfits? $5 \times 4 = 20$ Fun

3. Telephone numbers in the United States begin with three-digit area codes followed by seven-digit local telephone numbers. Area codes and local telephone numbers cannot begin with 0 or 1. How many different telephone numbers are possible? Fun

Fun $8 \cdot 10 \cdot 10 \cdot 8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ 6.4×10^8

$6,400,000,000$

4. How many different teams of 10 players can be chosen from a soccer team of 11?

Combo no order

$${}_{11}C_{10} = \frac{11!}{10!} = 11$$

Using combinations and permutations to find probability.

Ex 5. A teacher chooses 5 students at random from the names shown to work together on a group project. What is the probability that the 5 students' names begin with a consonant?

Total way ${}_{18}C_5$

cons. ${}_{13}C_5$ ↙ not

$$\frac{{}_{13}C_5 \cdot {}_5C_0}{{}_{18}C_5}$$

$$\frac{1287 \cdot 1}{8568} \approx .15 \quad 15\%$$

What is the probability that the 5 students' names end with a vowel? total ${}_{18}C_5$ (12)

$$\frac{{}_{12}C_5 \cdot {}_5C_0}{8568} = \frac{792}{8568} \approx .09 \quad 9\%$$

Ex6: Roman has a collection of 26 books-16 are fiction and 10 are nonfiction. He randomly chooses 8 books to take with him on vacation. What is the probability that he chooses 4 fiction and 4 nonfiction?

$$\frac{{}_{16}C_4 \cdot {}_{10}C_4}{{}_{26}C_8} = \frac{210 \cdot 1820}{15622275} = .2446$$

24.5%

How many 4 non-fiction possible: ${}_{10}C_4 = 210$

How many 4 fiction possible: ${}_{16}C_4 = 1820$

How many total possible:

${}_{26}C_8 = 15622275$